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INVESTIGATION OF SURFACE MOTION IN SOLIDS CAUSED BY REFLECTION OF AN INTENSE ACOUSTIC PULSE AT A PRESSURE RELEASE SURFACE.

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THESIS

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AT A PRESSURE RELEASE SURFACE

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Caused by Reflection of an Intense Acoustic Pulse

at a Pressure Release Surface

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ABSTRACT

This work reports theoretical investigations of the surface motion in solids caused by reflection of an intense acoustic pulse at a pressure release surface done using the surface velocity reflection coefficient. A formula was derived for the surface velocity reflection coefficient which relates the surface particle velocity for an incident compression wave to the particle velocities of the incident and reflected waves.

To obtain the surface profiles, two different assumptions were used. The first assumption was the conventional one. In this assumption, surface motion was neglected during the period that the pulse strikes the surface and it is called the fixed surface approximation. The second assumption is to consider the motion and distortions of the surface and is called the moving surface approximation.

Four different materials were investigated and the results were compared for different materials and in the two different approximations.

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TABLE OF SYMBOLS

c	velocity of propagation of waves in general.
c_1	velocity of dilatation waves in an unbounded medium.
c_2	velocity of distortion waves in an unbounded medium.
c_{11}, c_{12} ...	elastic constants in an aeolotropic medium.
E	Young's modulus.
k	bulk modulus.
$\hat{n}_1$, $\hat{n}_2$, $\hat{n}_3$	unit vectors.
R	coefficient of reflection.
$\mathcal{R}$	surface velocity reflection coefficient.
t	time.
v_ℓ	particle velocity within a longitudinal wave.
v_p	surface particle velocity.
v_t	particle velocity within a transverse wave.
u, v, w	displacement in x, y and z directions respectively.
$\bar{\alpha}$	angle of emergence.
Δ	dilatation.
ε	strain.
ε_{xx}, ε_{yy} ...	components of strain in Cartesian coordinates.
λ	Lamé's constant.
μ	rigidity modulus.
ν	Poisson's ratio.
ρ	density.
σ	stress.
σ_{xx}, σ_{xy} ...	components of stress in Cartesian coordinates.

$\bar{\omega}_x$, $\bar{\omega}_y$, $\bar{\omega}_z$ components of rotation in Cartesian coordinates.

∇^2 Laplacian operator.

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I. INTRODUCTION

Impulsive loads such as are produced by explosions and impacts may cause the failure of materials. An impulsive load is distinguishable from a conventional load primarily by its suddenness of application and its brevity of duration. Because of these loading characteristics, the reaction of a solid body to an impulsive load must be understood in terms of the propagation of waves in the body generated by the loading. If the loading is of sufficiently large amplitude, these waves produce extensive fracturing and large permanent distortions in the body upon which it acts. The phenomena associated with failure under impulsive loads are markedly different from those connected with failure under conventional loads. The reaction of materials to the extremely high, very transitory pressures imposed on them by impulsive loading is of great fundamental importance [Ref. 1].

The basic character of the fracture and plastic flow which are caused by impulsive loading is beginning to be understood [Ref. 1]. A common fracture process results by the generation of shear waves whose amplitude exceeds the shear strength of the material in which the wave is propagating. Such shear waves are usually generated by the reflection of a strong compressive wave from boundaries. Another common fracture process results when the pressure release wave generated by the reflection of the compressive wave from a boundary has too great an amplitude. A third common failure process involves the superposition of shocks reflected from various boundaries. Thus, understanding the way in which intense waves damage solids requires an understanding of reflection of waves from boundaries.

In the limited theoretical discussion which has been given such processes to date [Ref. 5], the reflection process is discussed using expression obtained for small amplitude waves. Experiments show that this discussion is sufficient for qualitative, but not always quantitative, understanding of some, but not all, of the observed phenomena.

The laws governing oblique reflection of elastic waves at a free surface are well known [Refs. 2, 3]. There is a particle motion associated with the reflection of these elastic waves that strike free surfaces obliquely. The object of this thesis is to study possible departures from the behavior predicted by the reflection formulae which result because of the finite amplitudes of waves involved by investigating the surface motion in solids caused by reflection of an intense acoustic pulse at a pressure release surface.

II. SURFACE MOTION DUE TO REFLECTION OF ELASTIC WAVES

A. ELASTICITY

Mathematical theories of elasticity are developed in the same general way. First, the notations of stress and strain are developed. Second, a stress-strain relationship between these quantities, or their derivatives, is assumed which idealizes the behavior of actual materials. Finally, using this relationship, equations of motion or equilibrium are set up which enable the state of stress or strain to be calculated when a body is subject to prescribed forces.

In general, the stress on a surface element in a solid body does not act normally to the surface. It has components both normal to the plane and tangential to it. In Fig. 2-1, the coordinate system has three mutually perpendicular axes 0_x, 0_y, 0_z. The stresses acting on three planes normal to these axes which pass through a point P are considered. There will be nine components of stress. These are denoted by σ_{xx}, σ_{yy}, σ_{zz}, σ_{xz}, etc. The first letter in the suffix indicates the direction of the stress and the second one defines the plane in which stress is acting. If an infinitesimal rectangular parallelepiped around P, with its faces normal to the axes, is considered, by taking moments, it may be seen that for equilibrium $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, and $\sigma_{yz} = \sigma_{zy}$. Thus there are only six independent components of stress. These completely define the stress at the point.

The strain of the medium is the relative displacement of the material particles in the medium. A rigid translation or rotation of a medium is not considered a strain. In order to find the strain at the point, its change in position relative to adjacent points must be analyzed.

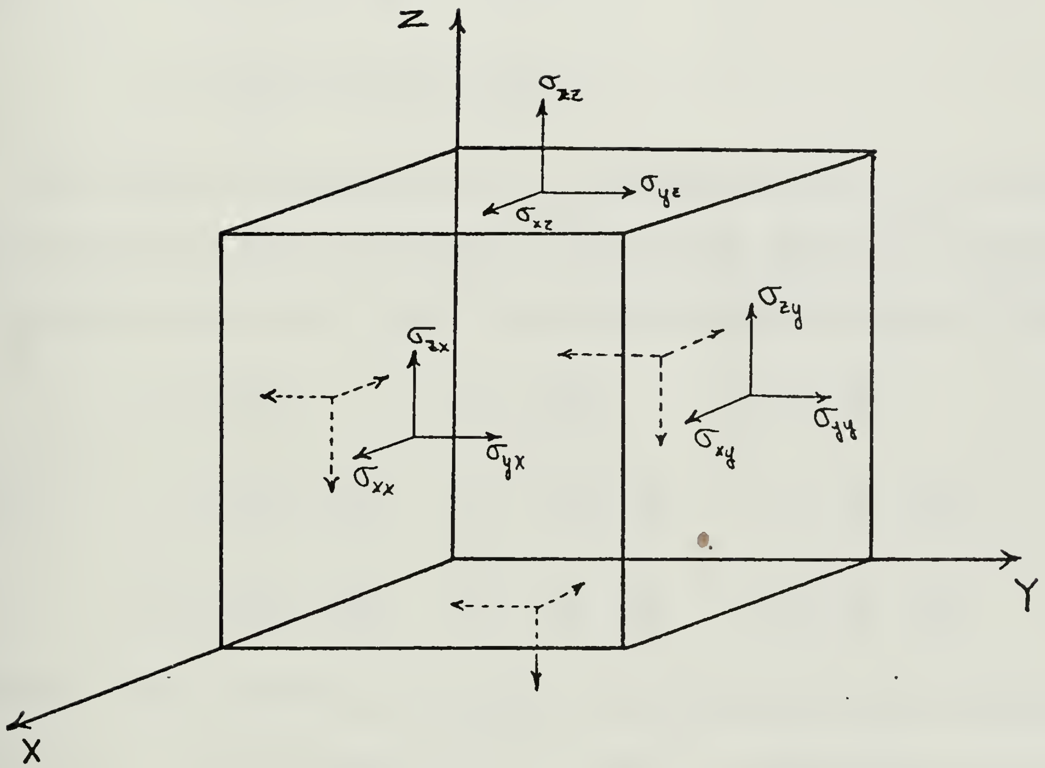

Figure 2.1. Stress components acting on an infinitesimal rectangular parallelepiped

Consider a point very close to P, which before displacement had coordinates $(x + dx)$, $(y + dy)$, $(z + dz)$. Let the displacement which it has undergone have components $(u + \delta u, v + \delta v, \omega + \delta\omega)$, where u, v and ω are the components of the displacement of point P. If δx, δy and δz are sufficiently small.

$$\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z$$

$$\delta v = \frac{\partial v}{\partial x}\delta x + \frac{\partial v}{\partial y}\delta y + \frac{\partial v}{\partial z}\delta z \tag{2-1}$$

$$\delta\omega = \frac{\partial\omega}{\partial x}\delta x + \frac{\partial\omega}{\partial y}\delta y + \frac{\partial\omega}{\partial z}\delta z$$

The relative displacement of all surrounding points P may be found if the values of the nine quantities $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial z}, \frac{\partial\omega}{\partial x}, \frac{\partial\omega}{\partial y}, \frac{\partial\omega}{\partial z}$ at the point are known. These nine quantities are regrouped as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial\omega}{\partial z}$$

$$\varepsilon_{yz} = \frac{\partial\omega}{\partial y} + \frac{\partial v}{\partial z} \qquad \varepsilon_{zx} = \frac{\partial u}{\partial z} + \frac{\partial\omega}{\partial x} \qquad \varepsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{2-2}$$

$$2\bar{\omega}_x = \frac{\partial\omega}{\partial y} - \frac{\partial v}{\partial z} \qquad 2\bar{\omega}_y = \frac{\partial u}{\partial z} - \frac{\partial\omega}{\partial x} \qquad 2\bar{\omega}_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

The first three quantities, ε_{xx}, ε_{yy}, ε_{zz} correspond to the fractional expansion. The second three ε_{yz}, ε_{zx} and ε_{xy} correspond to the components of shear strain in the planes denoted by their suffixes. The last three $\bar{\omega}_x$, $\bar{\omega}_y$ and $\bar{\omega}_z$ do not correspond to a deformation of the element around P, but are the components of its rotation as a rigid body.

Experimentally it is found that for most solids, the observed strains are proportional to the applied load, provided that the load does not exceed the elastic limit. This law is known as generalized form of

Hooke's law, and can be stated as: Each of the six components of stress is at any point a linear function of the six components of strain. This law may be written as

$$\begin{aligned}
\sigma_{xx} &= c_{11}\varepsilon_{xx} + c_{12}\varepsilon_{yy} + c_{13}\varepsilon_{zz} + c_{14}\varepsilon_{yz} + c_{15}\varepsilon_{zx} + c_{16}\varepsilon_{xy} \\
\sigma_{yy} &= c_{21}\varepsilon_{xx} + c_{22}\varepsilon_{yy} + c_{23}\varepsilon_{zz} + c_{24}\varepsilon_{yz} + c_{25}\varepsilon_{zx} + c_{26}\varepsilon_{xy} \\
\sigma_{zz} &= c_{31}\varepsilon_{xx} + c_{32}\varepsilon_{yy} + c_{33}\varepsilon_{zz} + c_{34}\varepsilon_{yz} + c_{35}\varepsilon_{zx} + c_{36}\varepsilon_{xy} \\
\sigma_{yz} &= c_{41}\varepsilon_{xx} + c_{42}\varepsilon_{yy} + c_{43}\varepsilon_{zz} + c_{44}\varepsilon_{yz} + c_{45}\varepsilon_{zx} + c_{46}\varepsilon_{xy} \\
\sigma_{zx} &= c_{51}\varepsilon_{xx} + c_{52}\varepsilon_{yy} + c_{53}\varepsilon_{zz} + c_{54}\varepsilon_{yz} + c_{55}\varepsilon_{zx} + c_{56}\varepsilon_{xy} \\
\sigma_{xy} &= c_{61}\varepsilon_{xx} + c_{62}\varepsilon_{yy} + c_{63}\varepsilon_{zz} + c_{64}\varepsilon_{yz} + c_{65}\varepsilon_{zx} + c_{66}\varepsilon_{xy}
\end{aligned} \tag{2-3}$$

The coefficients in these relations are known as the elastic constants of the material.

In an isotropic solid, the values of the coefficients are independent of the set of rectangular axes chosen. For such a solid, there are only two independent elastic constants, λ and μ known as Lamé's constants. All the elastic constants can be expressed in terms of the Lamé's constants. Thus, the Lamé's constants completely define the elastic behavior of an isotropic solid. For such a material, equation 2-3 may be written

$$\begin{aligned}
\sigma_{xx} &= \lambda\Delta + 2\mu\varepsilon_{xx} & \sigma_{yy} &= \lambda\Delta + 2\mu\varepsilon_{yy} & \sigma_{zz} &= \lambda\Delta + 2\mu\varepsilon_{zz} \\
\sigma_{yz} &= \mu\varepsilon_{yz} & \sigma_{zx} &= \mu\varepsilon_{zx} & \sigma_{xy} &= \mu\varepsilon_{xy}
\end{aligned} \tag{2-4}$$

Here $\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$, and is called dilatation. It represents the change in volume of a unit cube.

For convenience, four elastic constants are normally introduced. One is Young's modulus E, which is given by $\sigma_{xx}/\varepsilon_{xx}$ so that:

$$(2\text{-}5) \qquad E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$

A second is Poisson's ratio ν defined as $-\sigma_{yy}/\varepsilon_{xx}$ so that:

$$(2\text{-}6) \qquad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

A third is the bulk modulus k defined as the ratio between the applied pressure and the fractional change in volume when the solid is subjected to uniform hydrostatic compression so that:

$$(2\text{-}7) \qquad k = \lambda + \frac{2\mu}{3}$$

Finally, the rigidity of shear modulus, which is identical with Lamé's constant μ, is defined to be the ratio between the shear stress and the shear strain.

B. EQUATION OF MOTION IN AN ELASTIC MEDIUM

In order to obtain the equations of motion for an elastic medium, the variation in stress across a small parallelepiped with its sides parallel to a set of rectangular axes are considered (Fig. 2.2). The components of stress will vary across the faces; to obtain the force acting on each face, the value of stress at the center of each face times the area of the face has been taken.

Six separate forces will act parallel to each axis. The resultant force acting in the x-direction is

$$(\sigma_{xx} + \frac{\partial\sigma_{xx}}{\partial x}\delta x)\delta y\delta z - \sigma_{xx}\delta y\delta z +$$

$$+(\sigma_{xy} + \frac{\partial\sigma_{xy}}{\partial y}\delta y)\delta x\delta z - \delta_{xy}\delta x\delta z +$$

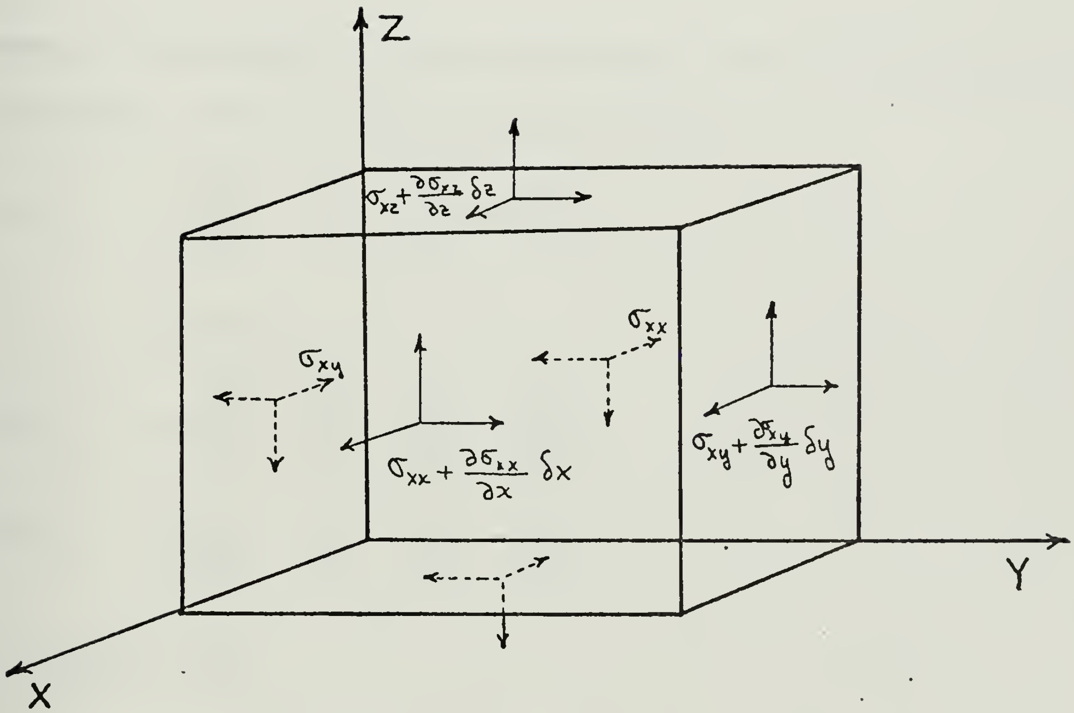

Figure 2.2. Stresses acting on a small rectangular parallelepiped

$$+ (\sigma_{xz} + \frac{\partial\sigma_{xz}}{\partial z}\delta z)\delta x\delta y - \sigma_{xz}\delta x\delta y$$

$$= (\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z})\delta x\delta y\delta z$$

By Newton's second law of motion, neglecting body forces such as gravity, this will be equal to

$$(\rho\delta x\,\delta y\delta z)\frac{\partial^2 u}{\partial t^2}$$

where ρ is the density of the element and u is the displacement in the x-direction. Thus the equations of motion are

(2-8) $$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z}$$

Similarly

(2-9) $$\rho\frac{\partial^2 v}{\partial t^2} = \frac{\partial\sigma_{yx}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z}$$

(2-10) $$\rho\frac{\partial^2 \omega}{\partial t^2} = \frac{\partial\sigma_{zx}}{\partial x} + \frac{\partial\sigma_{zy}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z}$$

C. ELASTIC WAVES

Equations 2-8, 2-9 and 2-10 will hold whatever the stress-strain behavior of the medium. By substituting the equations 2-4 for the stress components in Equation 2-8, one obtains

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(\lambda\Delta + 2\mu\varepsilon_{xx}) + \frac{\partial}{\partial y}(\mu\varepsilon_{xy}) + \frac{\partial}{\partial z}(\mu\varepsilon_{xz})$$

By using the equations 2-2, one finds

(2-11) $$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\frac{\partial\Delta}{\partial x} + \mu\nabla^2 u$$

Similarly

(2-12) $$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda + \mu)\frac{\partial\Delta}{\partial y} + \mu\nabla^2 u$$

(2-13) $$\rho\frac{\partial^2 \omega}{\partial t^2} = (\lambda + \mu)\frac{\partial\Delta}{\partial z} + \mu\nabla^2 \omega$$

Equations 2-11 through 2-13 are the equations of motion of an isotropic, elastic solid in which body forces are absent. They predict the propagation of two types of waves through the medium. If both sides of the above equations are differentiated with respect to x, y and z respectively, they become

(2-14) $$\rho\frac{\partial^2 \Delta}{\partial t^2} = (\lambda + 2\mu)\nabla^2 \Delta$$

This is the wave equation and shows that the dilatation Δ is propagated through the medium with velocity

(2-15) $$c_1 = [(\lambda + 2\mu)/\rho]^{\frac{1}{2}}$$

On the other hand, if Δ is eliminated, it can be shown that

(2-16) $$\rho\frac{\partial^2 \bar{\omega}_x}{\partial t^2} = \mu\nabla^2 \bar{\omega}_x$$

where $\bar{\omega}_x$ is the rotation about the x-axis. Similar equations may be obtained for $\bar{\omega}_y$ and $\bar{\omega}_z$. Thus the rotation is propagated with velocity

(2-17) $$c_2 = (\mu/\rho)^{\frac{1}{2}}$$

This shows that in the interior of any elastic solid waves may be propagated with two different velocities. These two types of waves should be termed dilatation waves and distortion waves.

Any plane wave propagated through on isotropic elastic medium must travel one or the other of the above velocities. The waves which propagate with velocity c_1 are called longitudinal waves. The motion is along the direction of propagation. The other type waves, which propagate with velocity c_2, are called transverse waves since the motion is parallel to the wave front.

The equations 2-14 and 2-16 are of the form

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \nabla^2 \psi \tag{2-18}$$

When the deformation is a function of only one coordinate, e.g. x, the equation becomes:

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2} \tag{2-19}$$

The general solution of this is:

$$\psi = f(x - ct) + F(x + ct)$$

where f and F are arbitrary functions depending on initial conditions, f corresponding to a plane wave travelling along the positive direction of the x-axis, and F to one in the direction opposite to this.

D. REFLECTION AT PRESSURE RELEASE SURFACE

Normal incidence. Any elastic wave will be reflected when it reaches a free surface of the material in which it is travelling. The simplest case occurs when the wave strikes the surface normally. In a longitudinal wave, since the stress normal to the surface at the surface must be zero, the reflected pulse must be opposite in sense

to the incident pulse (compression reflected as tension and vice versa).

Oblique incidence. When an elastic wave strikes a free surface obliquely, the situation is much more complicated. This complication arises from the fact that the energy of incident wave is partitioned into two reflected waves instead of only one. When either a longitudinal or a transverse pulse strikes a surface obliquely, both a longitudinal and a transverse wave are reflected. (Figure 2.3).

Consider first a longitudinal wave striking a surface obliquely. There will be a reflected longitudinal portion in which the angle of incidence α is equal to the angle of reflection. There will also be a transverse wave which is generated at the point of reflection. From Snell's law, the relationship between the angle of incidence α and the angle of reflection β is

$$\text{(2-20)} \qquad \frac{\operatorname{Sin} \alpha}{\operatorname{Sin} \beta} = \frac{c_1}{c_2} = [2(1 - \nu)/(1 - 2\nu)]^{\frac{1}{2}}$$

From equation 2-20, it is seen that the angle of reflection β is the function of the incidence angle and the Poisson's ratio of the material. Figure 2.4 shows β as a function of α for several values of Poisson's ratio.

This generation of a shear wave can be visualized in terms of particle velocities (Fig. 2.5). It can also be seen that a shear wave generated by the oblique reflection of a longitudinal wave is polarized in the plane of incident; that is, the particle velocity associated with this shear wave is normal to the direction of propagation and directed in only one direction. The case of a compression

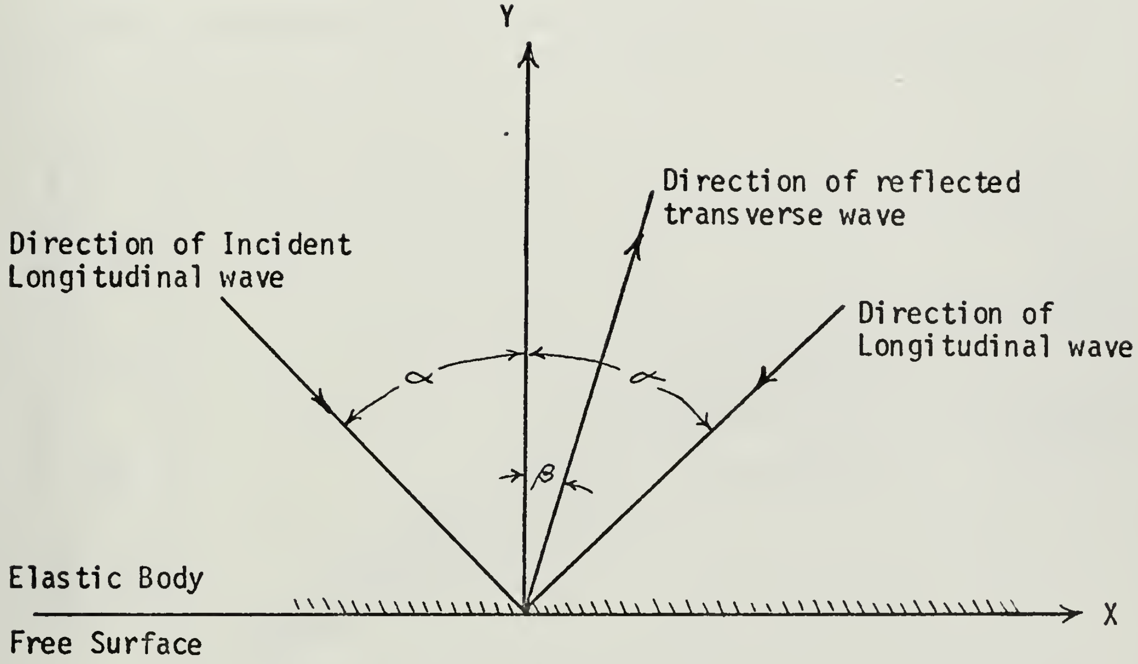

Figure 2.3 Oblique incidence of a plane longitudinal wave at a free surface

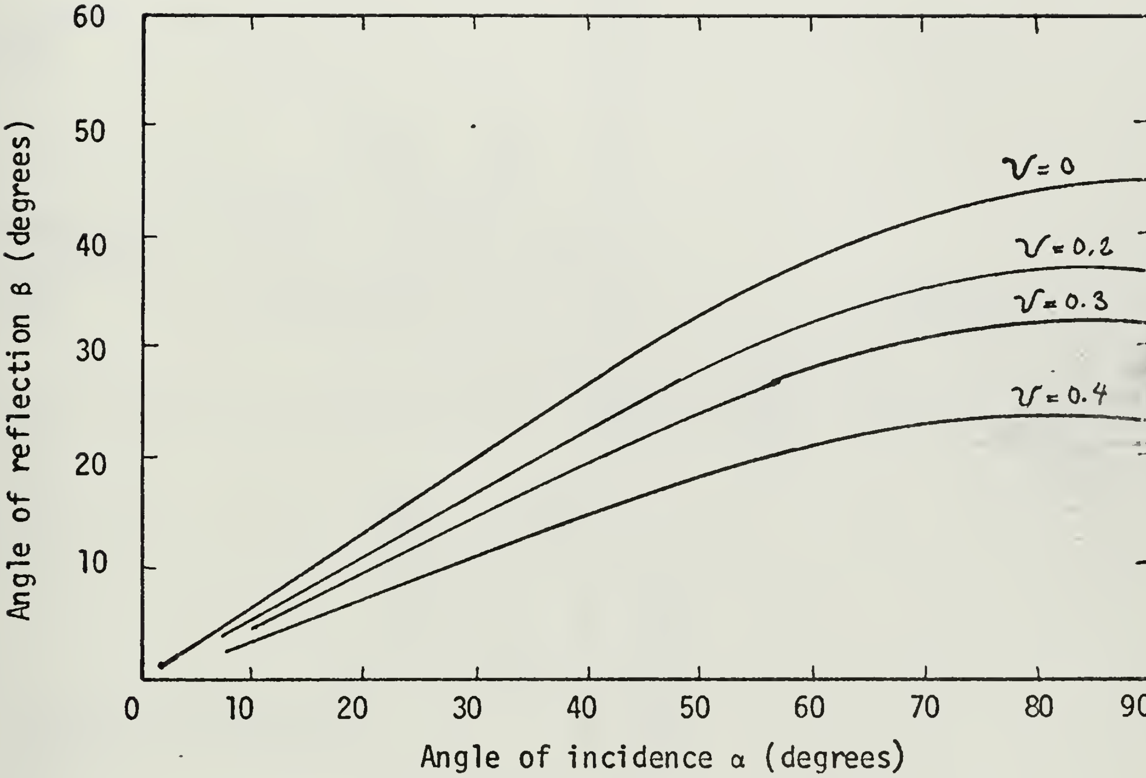

Figure 2.4. Dependence of Angle of Reflection of Generated Shear Wave upon Poisson's Ratio and Angle of Incidence of Longitudinal Wave

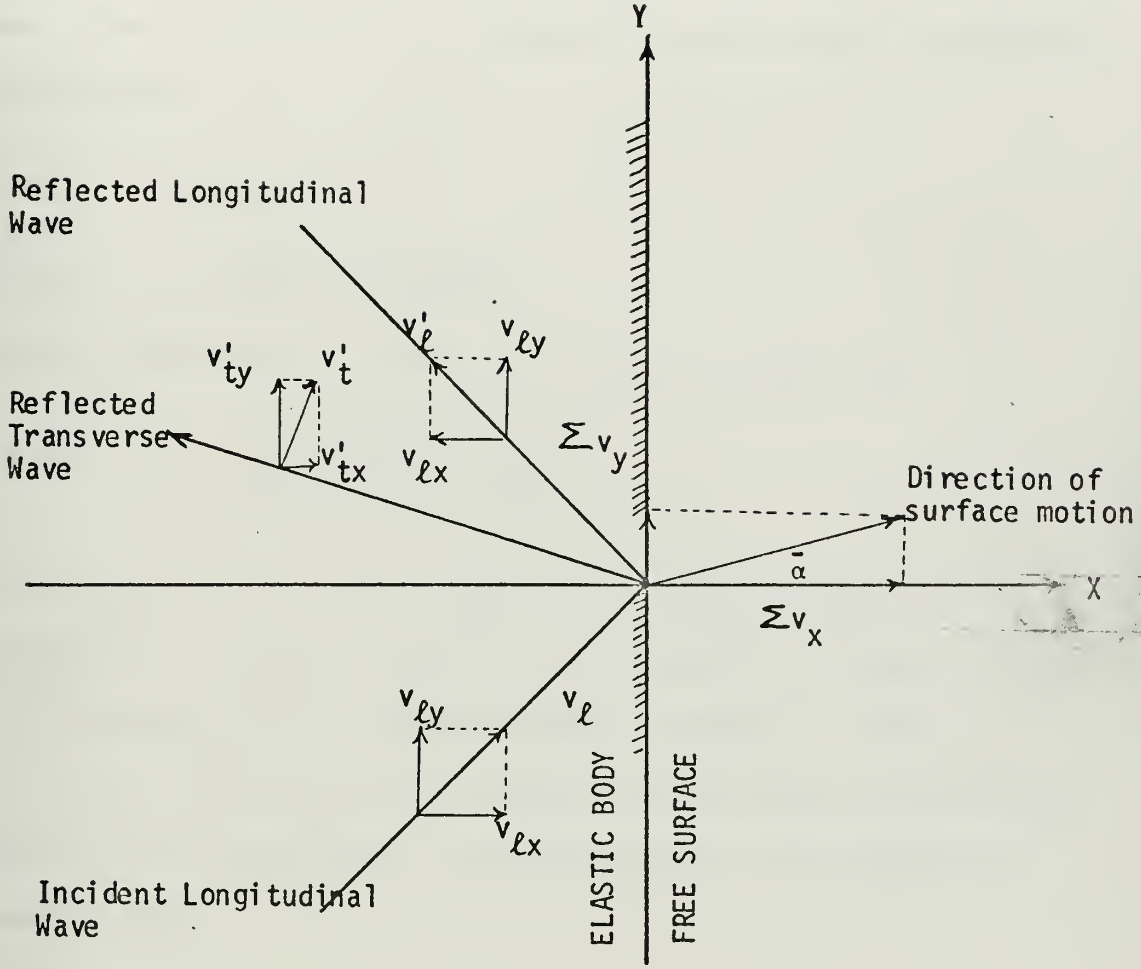

Figure 2.5. Particle Motion Associated with an obliquely incident plane longitudinal wave

wave striking the free surface of a semi-infinite body will be considered. Let σ be the instantaneous value of normal stress of the incident wave, σ' the corresponding normal stress of the reflected longitudinal wave and τ' the shear stress of the induced transverse wave. These stresses are all related by a coefficient of reflection R such that [Ref. 3]

(2-21) $$\sigma' = R\sigma$$

(2-22) $$\tau' = [(R + 1)\ \mathrm{Cot}2\beta]\sigma$$

and the coefficient of reflection R is given by [Ref. 3].

(2-23) $$R = \frac{\tan\beta\ \tan^2 2\beta - \tan\alpha}{\tan\beta\ \tan^2 2\beta + \tan\alpha}$$

R is a function of α and β. Since R is a function of β, it is actually a function of the Poisson's ratio of the material. Figure 2.6 shows R as a function of α for several values of Poisson's ratios.

A very similar series of statements can be made for the oblique reflection of a shear wave. By referring back to R as defined by equation 2-23.

(2-24) $$\tau' = R\tau$$

(2-25) $$\sigma' = [(R - 1)\tan^2\beta]\tau$$

where τ, τ' and σ' are the instantaneous values of incident shear stress, generated shear stress and generated longitudinal stress, respectively, β now representing the angle of incidence.

When an incident wave strikes a free surface obliquely, the surface will be displaced such that the angle of emergence $\bar{\alpha}$ satisfies [Ref. 3, 4].

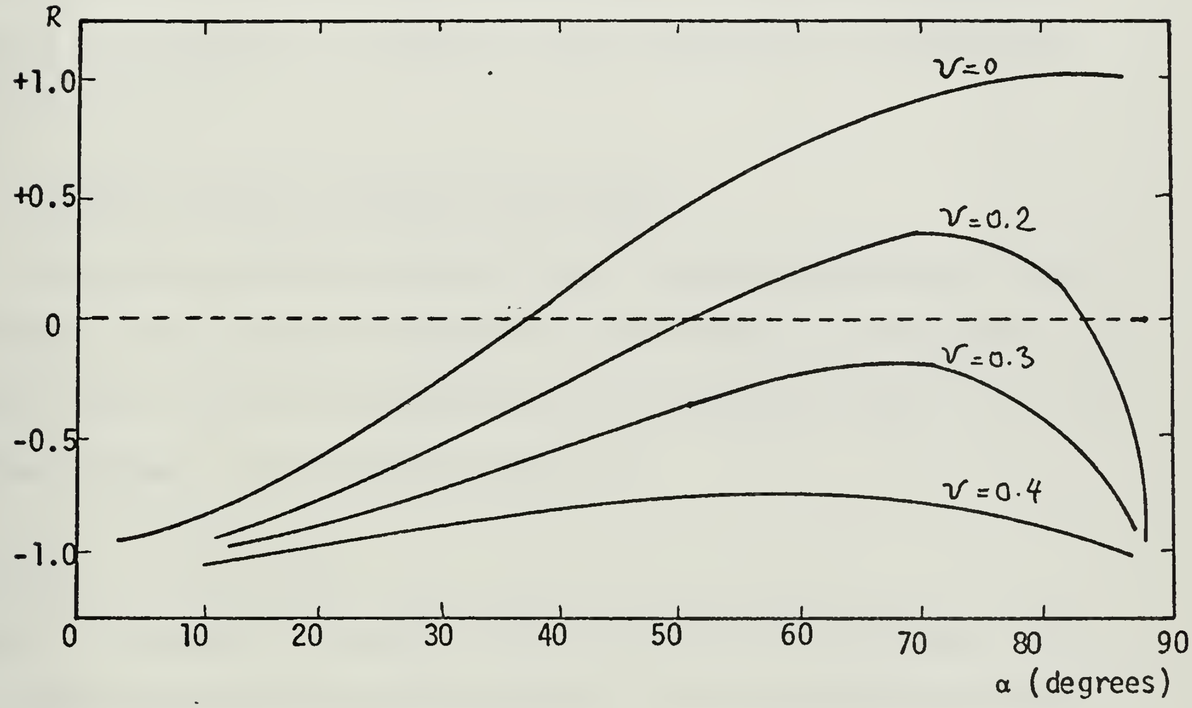

Figure 2.6. Coefficient of Reflection as a Function of Angle of Incidence for Several Values of Poisson's Ratio

(2-26) $\bar{\alpha} = 2\beta$

The angle of emergence is also related to rectilinear components of the particle velocity by

(2-27) $\bar{\alpha} = \tan^{-1}(v_x/v_y)$

where v_x is the sum of the particle velocity components perpendicular to the surface, v_y is the sum of the particle velocity vectors parallel to the surface as seen in Fig. 2.5. Again it can be seen that the angle of emergence is a function of both the angle of incidence and Poisson's ratio.

E. SURFACE VELOCITY REFLECTION COEFFICIENT

The surface particle velocity for an incident compression wave is related to the particle velocities of the incident and reflected waves. For a plane elastic longitudinal wave, the particle velocity ν_ℓ at any time is given by the expression [Ref. 4]

(2-28) $\nu_\ell = \sigma/\rho c_1$

where σ represents the normal stress associated with the wave. For a compressive wave the particle velocity is in the same direction as the velocity of wave propagation; for a tension wave the particle velocity is directed oppositely from that of propagation of the wave. A corresponding relationship exists for particle velocity ν_t within a transverse wave, namely,

(2-29) $\nu_t = \tau/\rho c_2$

where τ is the shear stress associated with the transverse wave.

From Fig. 2.5, the surface particle velocity is

$$(2\text{-}30) \qquad \vec{v}_p = \vec{v}_\ell + \vec{v}'_\ell + \vec{v}'_t$$

By substituting equations 2-28 and 2-29 into the equation 2-30

$$(2\text{-}31) \qquad \vec{v}_p = \frac{1}{\rho c_1} (\sigma \hat{n}_1 + \sigma' \hat{n}_2 + \frac{c_1}{c_2} \tau' \hat{n}_3)$$

where $\hat{n}_1$ is the unit vector in the direction parallel to the incident longitudinal wave, $\hat{n}_2$ is the unit vector in the direction parallel to the reflected longitudinal wave and $\hat{n}_3$ is the unit vector in the direction perpendicular to the reflected transverse wave. By substituting equations 2-21 and 2-22 into the equation 2-31

$$(2\text{-}32) \qquad \vec{v}_p = \frac{\sigma}{\rho c_1} [\hat{n}_1 + \hat{n}_2 R + \hat{n}_3 \frac{c_1}{c_2} (R + 1) \mathrm{Cot}\ 2\beta]$$

or

$$(2\text{-}33) \qquad \vec{v}_p = \vec{R}\sigma$$

where

$$(2\text{-}34) \qquad \vec{R} = \frac{1}{\rho c_1} [\hat{n}_1 + \hat{n}_2 R + \hat{n}_3 \frac{c_1}{c_2} (R + 1) \mathrm{Cot}\ 2\beta]$$

From Fig. 2.5

$$\hat{n}_1 = \hat{i}\ \mathrm{Cos}\ \alpha + \hat{j}\ \mathrm{Sin}\ \alpha$$

$$\hat{n}_2 = -\hat{i}\ \mathrm{Cos}\ \alpha + \hat{j}\ \mathrm{Sin}\ \alpha$$

$$\hat{n}_3 = \hat{i}\ \mathrm{Sin}\ \beta + \hat{j}\ \mathrm{Cos}\ \beta$$

By substituting these equations into equation 2-34, it can be shown that

$$\vec{R} = \frac{1}{\rho c_1}\left\{\hat{i}[(1 - R)\ \mathrm{Cos}\ \alpha + \frac{c_1}{c_2}(1 + R)\ \mathrm{Cot}\ 2\beta\ \mathrm{Sin}\ \beta] + \right.$$
$$\left. + \hat{j}[(1 + R)(\mathrm{Sin}\ \alpha + \frac{c_1}{c_2}\ \mathrm{Cot}\ 2\beta\ \mathrm{Cos}\ \beta)]\right\} \tag{2-35}$$

On the other hand

$$\vec{v}_p = v_p(\hat{i}\ \mathrm{Cos}\ 2\beta + \hat{j}\ \mathrm{Sin}\ 2\beta) \tag{2-36}$$

By using equations 2-33, 2-35 and 2-36, it can be shown that

$$R = \frac{(1 + R)}{\rho c_2}\left(\frac{\mathrm{Cot}\ 2\beta\ \mathrm{Cos}\ \beta + \mathrm{Sin}\ \beta}{\mathrm{Sin}\ 2\beta}\right) \quad \text{and} \tag{2-37}$$

$$v_p = R\sigma$$

where R is called the surface velocity reflection coefficient. From this formula, it can be seen that the surface velocity reflection coefficient is a function of the angle of incidence, the Poisson's ratio and the density of the material.

III. INVESTIGATION OF SURFACE MOTION

A. NATURE OF INVESTIGATION

The theories of oblique reflection of elastic waves at a free surface were outlined in the previous chapter. Such waves cause a particle motion on the surface. The motion of the surface will cause the reflection characteristics of an acoustic pulse to change as various portions of the pulse encounter to surface. The theoretical investigations of the surface motion in solid caused by reflection of an intense acoustic pulse at a pressure release surface were facilitated by using the surface velocity reflection coefficient R which was derived in equation 2-5.

The intent was to illustrate the surface motion for different materials, for different loadings, for different angles of incidence. For this purpose the IBM-360 computer was used. The surface profiles were obtained by using two different approaches. The first approach was the conventional one which is called the fixed surface approximation. This assumes that surface motion can be neglected during the period that the pulse strikes the surface. The second approach is consider the motion and distortions of the surface. This is called the moving surface approximation. The results of these two approximations were then compared.

The computer programs used in this investigation are listed in Appendix A. The fixed surface approximation computes the surface distortion by simply integrating the surface velocity, obtained by using a constant surface velocity reflection coefficient, resulting from a pulse arriving at the $x = 0$ plane. The moving surface approximation makes two major corrections. First, the surface velocity is

computed at each time from the displaced position of the surface, the displacement resulting from the previous portions of the pulse. Secondly, account was taken of changes in the angle of incidence caused by distortions of the surface from a plane. This was done by constructing the normal to an equation describing the surface obtained by passing a quadratic curve through the point in question and its neighboring points. Because of the time and position dependent changes in the pulse shape, numerical integration was used to convert the surface velocity into surface displacement.

Throughout this investigation a saw-toothed shape stress pulse with 1 mm. pulse width has been used to illustrate the surface motion. The amplitude of the stress was changed from a value equal to the normal fracture stress to ten times this quantity. The normal fracture stress is the maximum tensile stress which the material will tolerate. A more exact definition would be the minimum dynamic tensional stress required to rupture the material. The angle of incidence was varied from 5 to 85 degrees, using 5-degree increments. Four materials, aluminum, steel, plexiglass, and lead, were investigated. The characteristics of these materials are given in Table I.

B. RESULTS

<u>Comparison of Approximations</u>: As an illustration of the differences between the two approximations, the surface motion produced in aluminum and lead for a sawtooth compressive pulse of amplitude equal to the fracture stress incident at 30° computed in the two approximations is illustrated in Figs. 3.1 and 3.2. The maximum displacement produced by the pulse is the same in the two approximations. This result is

	Plexiglass	Aluminum	Steel	Lead
Normal Fracture Stress	1.1 K bar.	9.6 K bar.	11.6 K bar.	1.0 K bar.
Density	1.18 gr/cm^3	2.7 gr/cm^3	7.8 gr/cm^3	11.34 gr/cm^3
Velocity of Longitudinal Wave	2800 m/sec.	6374 m/sec.	5960 m/sec.	2160 m/sec.
Velocity of Transverse Wave	1350 m/sec.	3111 m/sec.	3235 m/sec.	700 m/sec.
Poisson's Ratio	0.34	0.13	0.28 - 0.30	0.40

TABLE I

SOME PHYSICAL PARAMETERS FOR MATERIALS INVESTIGATED

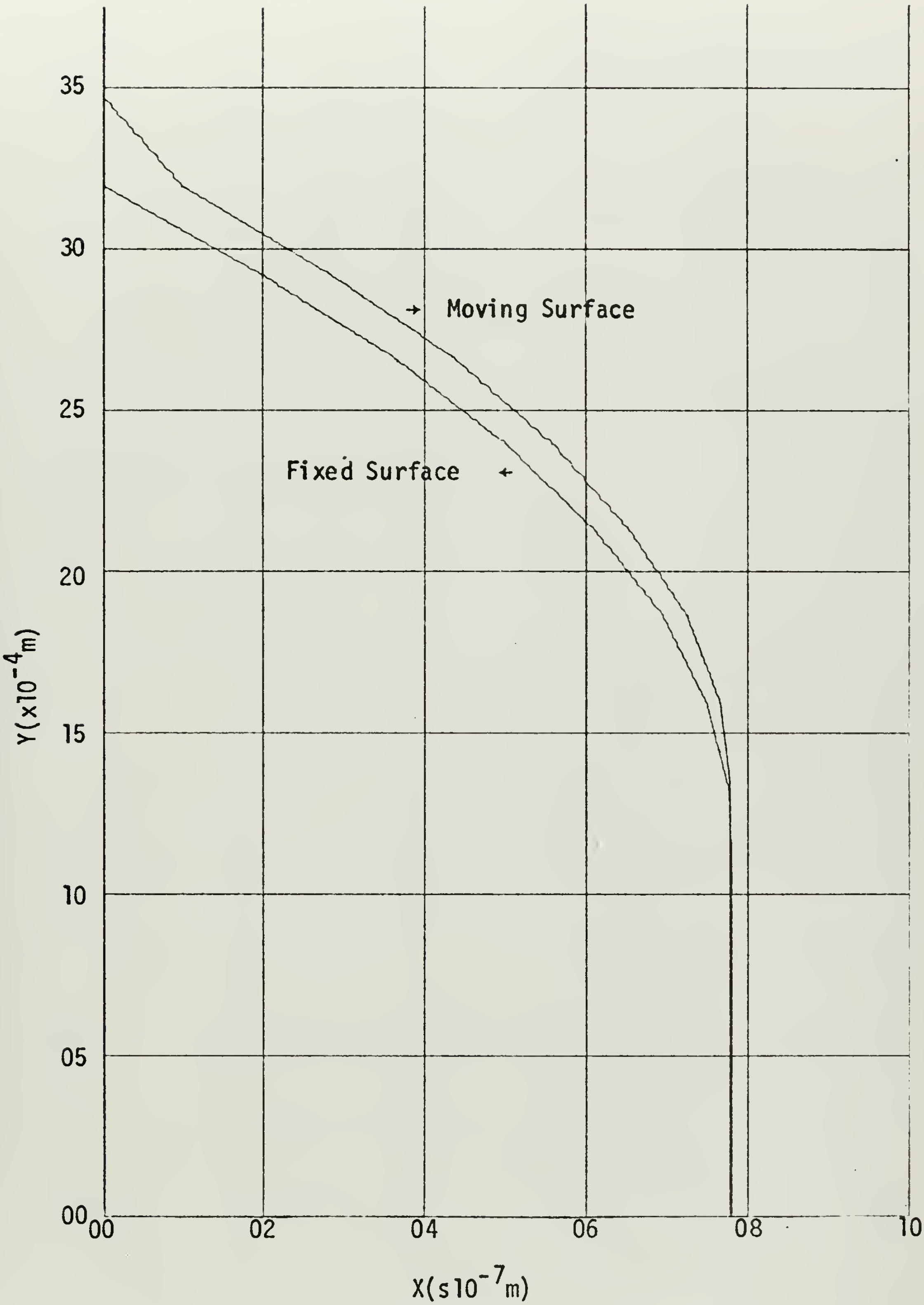

Figure 3.1. Surface motions for aluminum under two different assumptions. Amplitude of stress is equal to normal fracture stress, α = 30°, time = 0.251 x 10^{-6} sec.

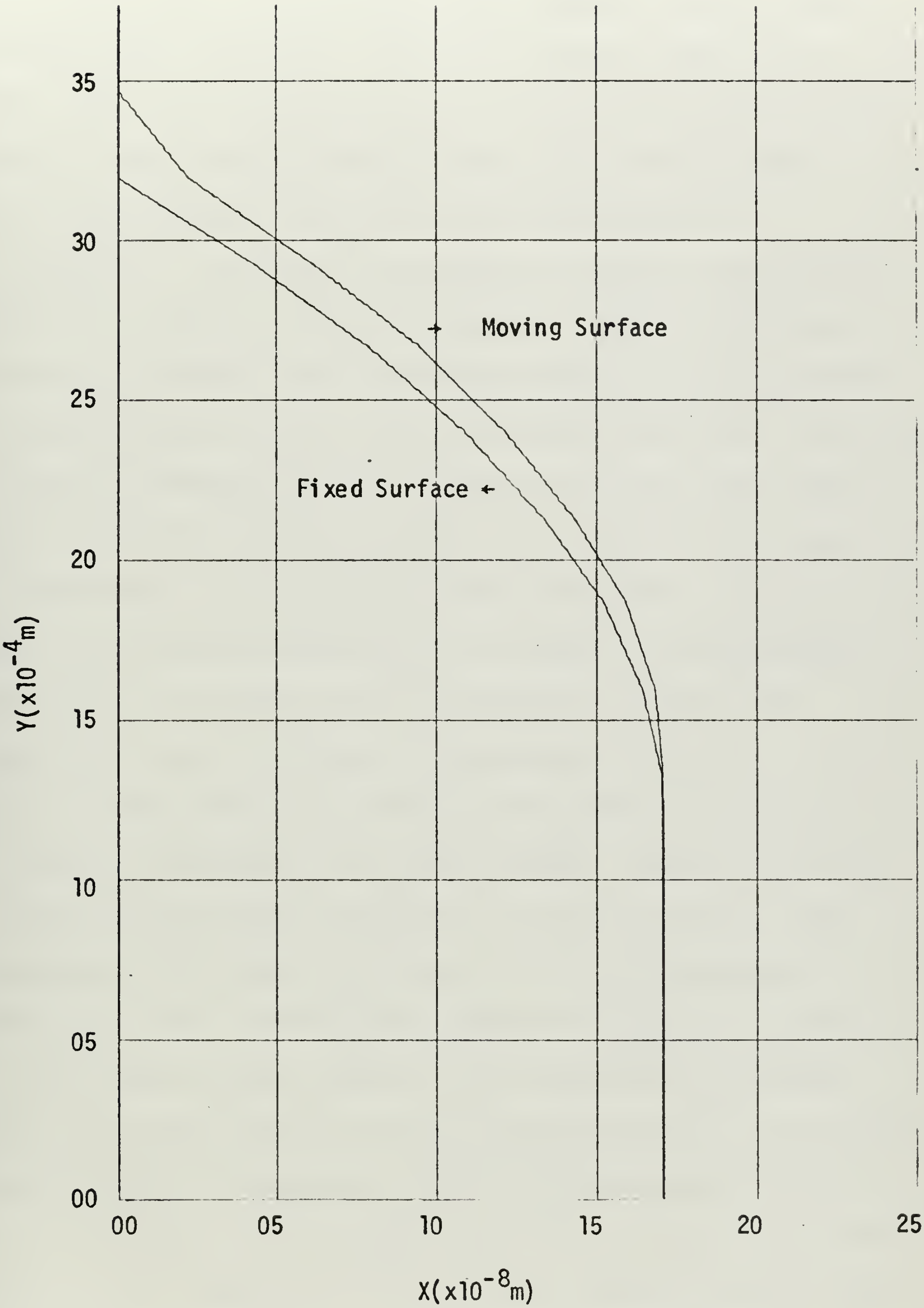

Figure 3.2. Surface motions for lead under two different assumptions. Amplitude of stress is equal to normal fracture stress, $\alpha = 30°$, time = 0.741×10^{-6}

expected since the net displacement can be computed from the impulse contained within the incident pulse and the elastic moduli of the medium within which it propagates. The moving surface approximation predicts a more rapid displacement of the surface than the fixed surface approximation. This also is quite understandable.

<u>The Angle ϕ</u>: The change of the angle of incidence due to surface distortion is called ϕ. During the motion of surface ϕ at any given spot changes with time. To illustrate the change in ϕ by the time at a given position, the curve is drawn for aluminum and given in Fig. 3.3. It shows that the angle ϕ starts to increase after the pulse strikes to the surface, and reaches its maximum value, then decreases again. But the decrease is relatively slower than the increase. This is a result of the assumed sawtooth pulse. The angle ϕ is also effected by the angle of incidence and by the amplitude of stress. The effect of the angle of incidence is illustrated in Fig. 3.4. Figure 3.4 shows the change in angle ϕ by the angle of incidence for all materials investigated. The angle ϕ changes linearly with stress amplitude because of using the Hooke's Law and the relatively small displacements. The change in the angle ϕ with stress amplitude is given in Fig. 3.5. In this figure, aluminum is chosen as an example.

In the moving surface assumption, the distortion of the surface from a plane effects the pulse shape and causes distortion on the pulse. Assuming plane wave propagation, there is no distortion at the front and tail end of the reflected pulse since it is reflected from a flat surface. But in between these two ends, the different portions of the pulse cause different distortions on the surface which result in different angle of correction ϕ. Because of different ϕ, the reflected

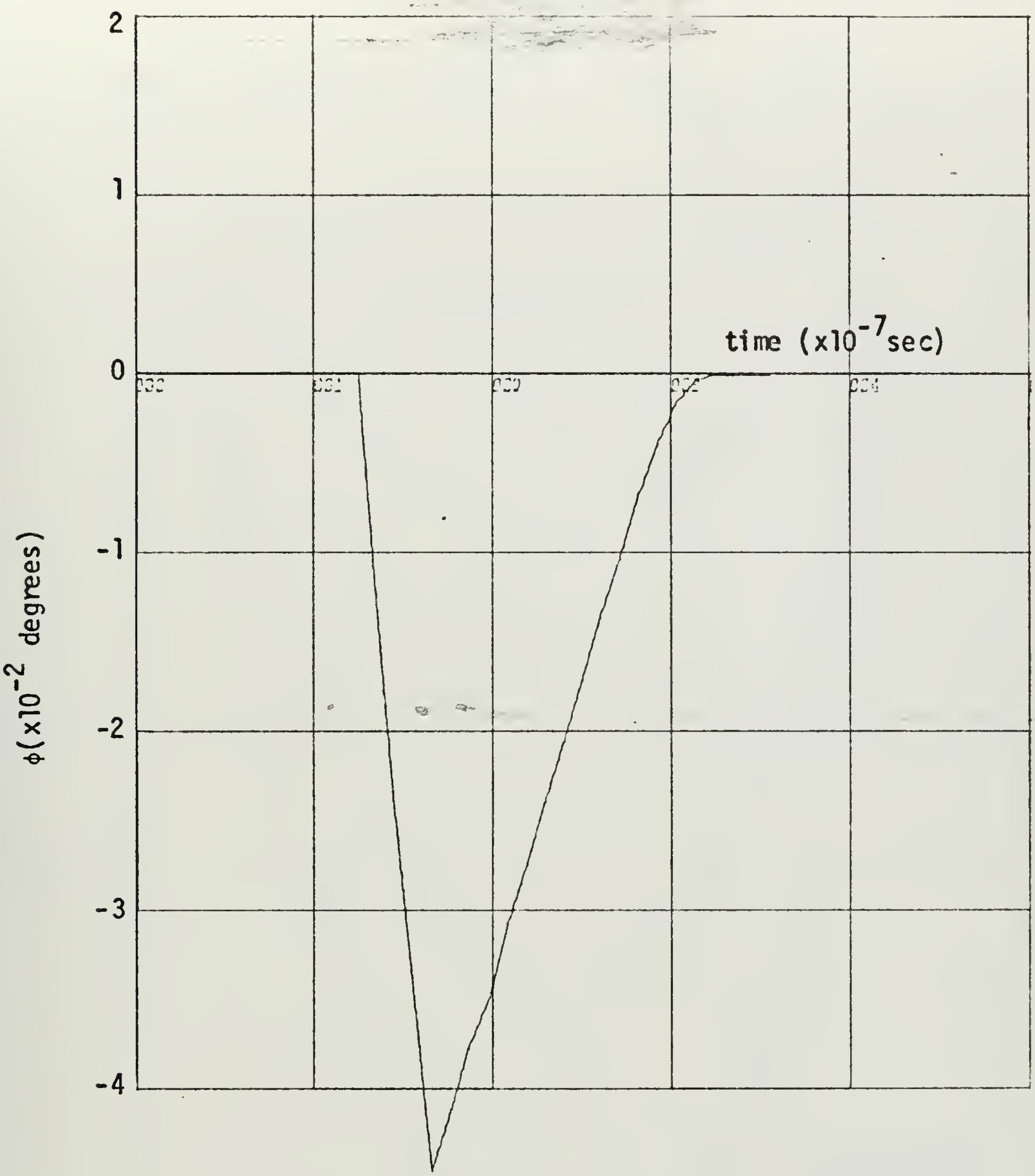

Figure 3.3. Change in angle ϕ with time for aluminum. Stress amplitude equals to normal fracture stress.

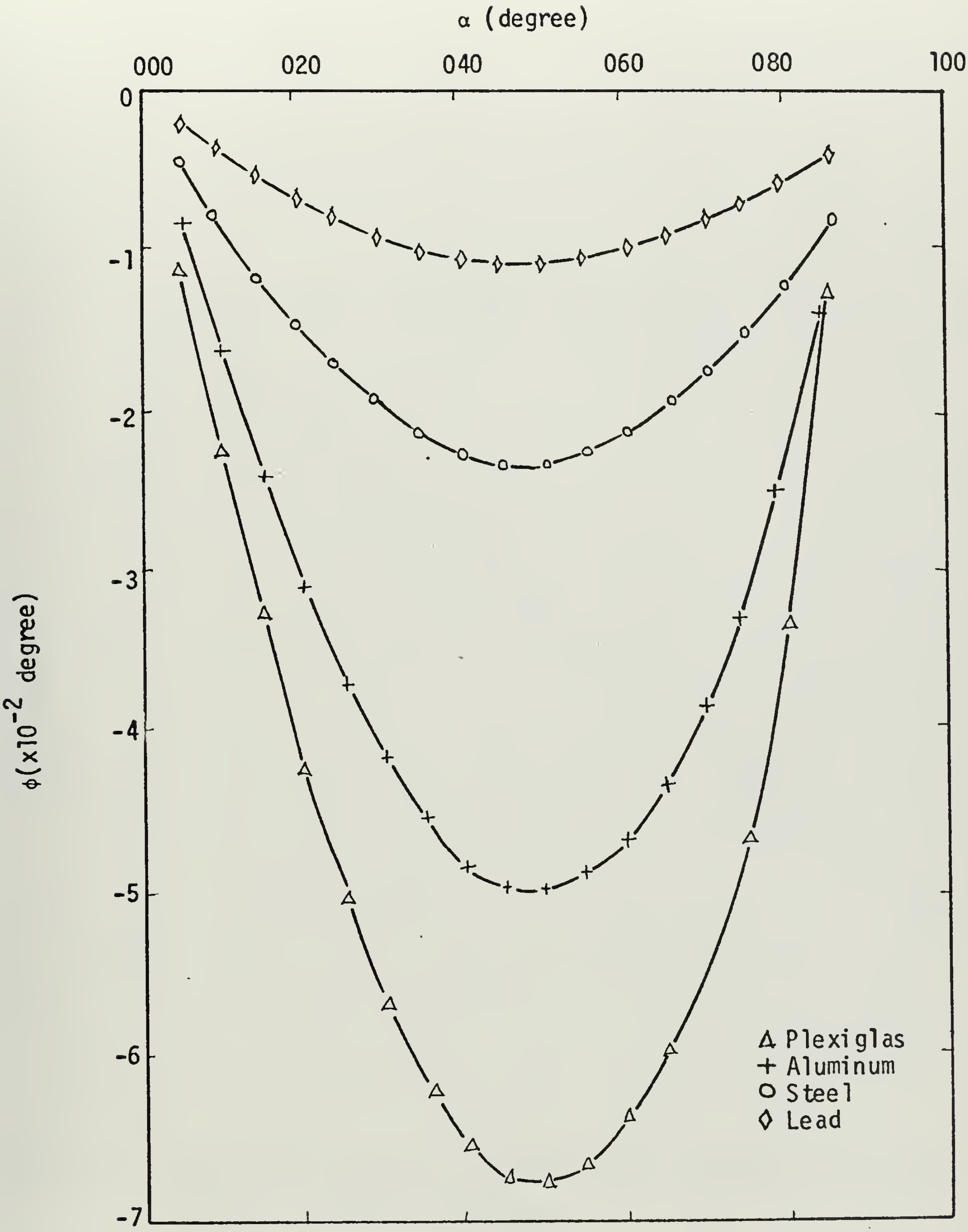

Figure 3.4. The change in angle ϕ with angle of incidence α for different materials. The amplitude of pulse is normal fracture stress.

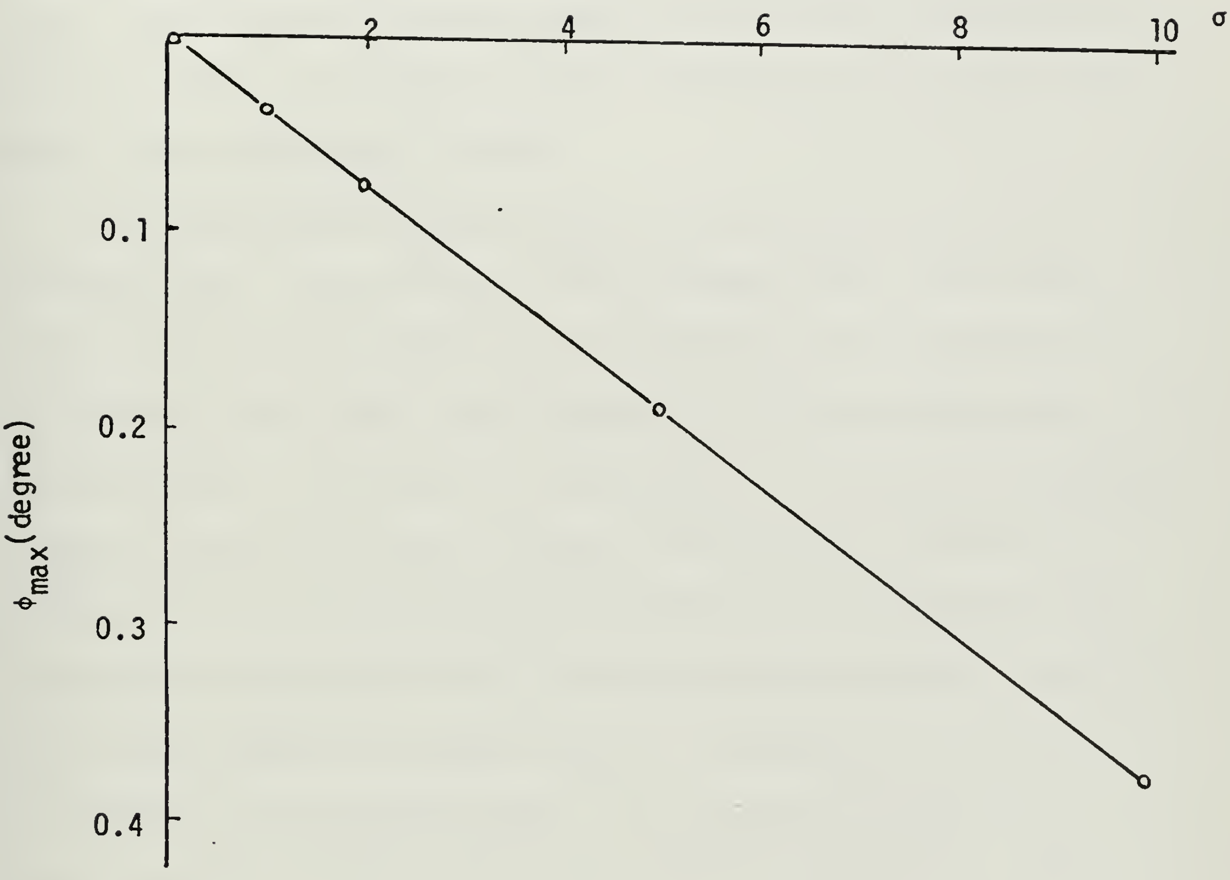

Figure 3.5. The change in angle ϕ_{max} with stress amplitude for aluminum. $\alpha = 60°$, $t = 0.251 \times 10^{-6}$ sec.

angle is not the same as the instant angle of incidence for these portions of pulse. Thus the wave fronts for these portions of pulse are not in the same direction relative to the front and tail end of the pulse. Hence the reflected pulse under the moving surface assumption is not the same as the incident pulse. From the investigation, it was seen that the pulse became far apart after the reflection from the surface. And this causes the distortion in the stress field which is not investigated in here.

The moving surface assumption also effects the amplitude of the reflected pulse. The amplitude of the reflected stress pulse equals to the product of the coefficient of reflection and the amplitude of the incident stress pulse. But from Fig. 2.6, it can be seen that the coefficient of reflection is the function of the angle of incidence. Since the assumption causes a change in the angle of incidence, the amplitude of the reflected pulse is not the same as the amplitude of the reflected stress pulse under the fixed surface assumption condition. It is either weaker or stronger than it. It depends on the angle of incidence and the material used. To illustrate the effect of ϕ on the amplitude of stress pulse, two numerical examples were done for aluminum. The coefficients of reflection were calculated by using the equation 2-23 for two different angles of incidence, 30 and 40 degrees. The amplitude of stress was the normal fracture stress and the maximum angles of correction were used in both cases. For 30 degrees angle of indidence, by using fixed surface approximation, the coefficient of reflection equals -0.778. But in the same condition by using the moving surface approximation, it equals -0.790. For

45 degrees angle of incidence, the first approximation results in the coefficient of reflection equals to -0.579, the second approximation results in the coefficient of reflection equals to -0.590.

Surface Displacement: Surface displacement is effected by the stress amplitude and by the angle of incidence. The surface displacement increases linearly with stress amplitude. However, it decreases by increasing the angle of incidence. The surface motions for two different materials, aluminum and lead, are illustrated in Figs. 3.6 and 3.7.

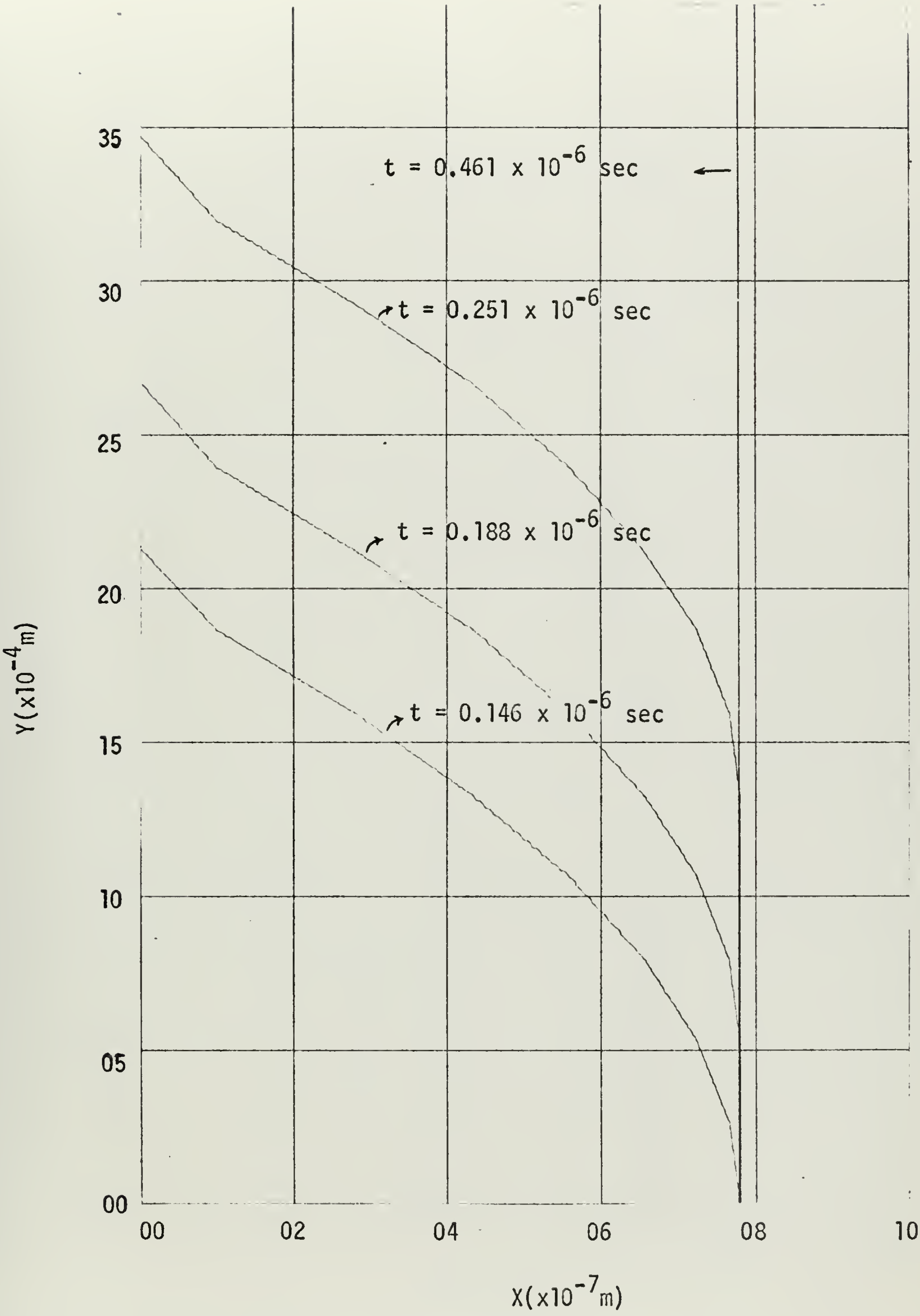

Figure 3.6. Surface motions for aluminum. Stress amplitude equals to normal fracture stress and α = 30.

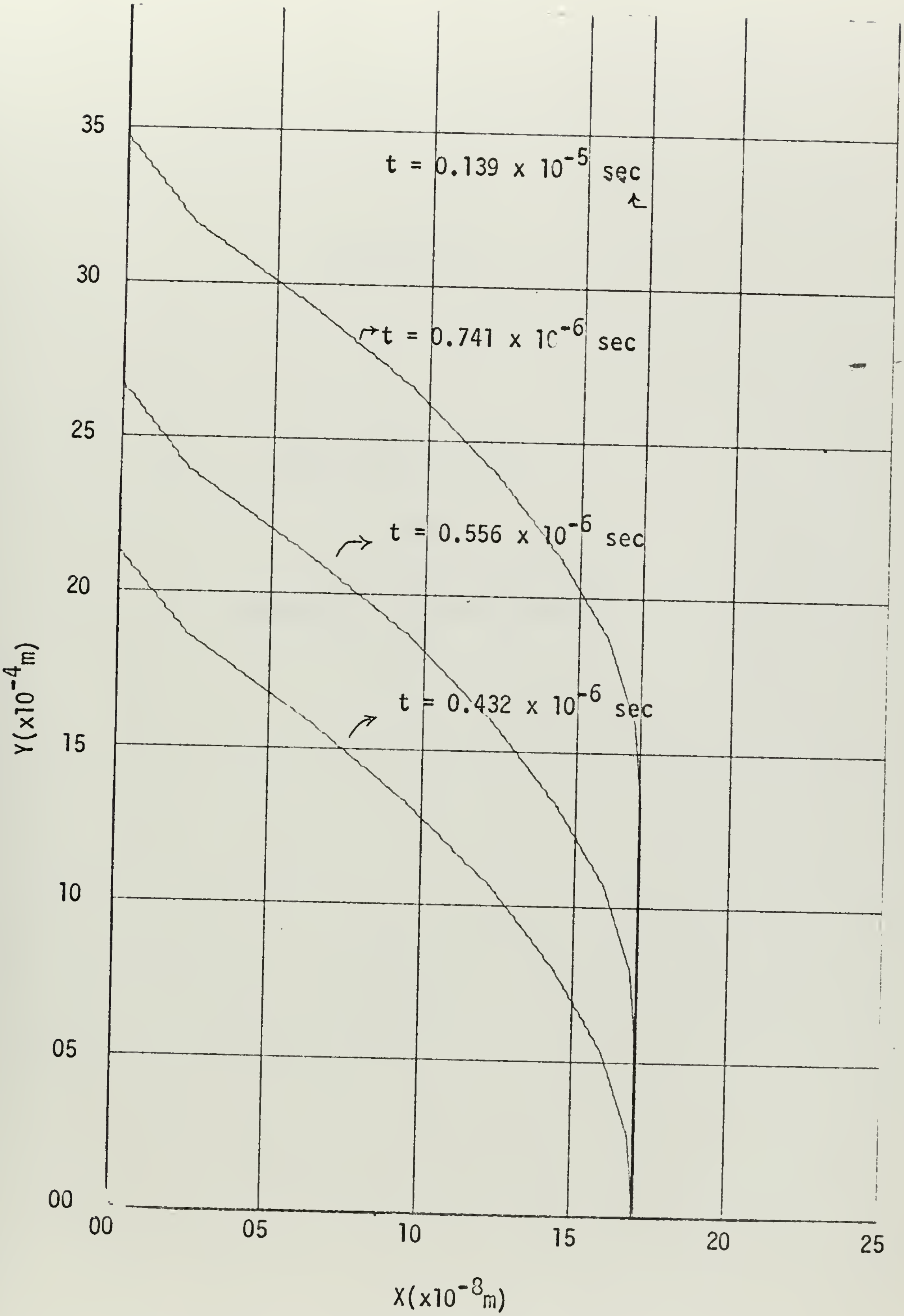

Figure 3.7. Surface motions for lead. Stress amplitude equals to the normal fracture stress, $\alpha = 30°$.

APPENDIX A

THE COMPUTER PROGRAMS

FOR

SIMULATION OF SURFACE MOTIONS

```
C     SURFACE MOTION BY USING MOVING SURFACE ASSUMPTION
      DIMENSION TIME(2000),ALFA(20),X(18000),Y(18000)
C     READ FHYSICAL PARAMETERS OF MATERIAL
      READ(5,15) PI,PL,RO,SIG,SL,ST
   15 FORMAT(6E11.5)
C     INCREMENT IN TIME AND IN ANGLE OF INCIDENCE
      DELT=PL/(15.0*SL)
      DELA=PI/36.0
      ALFA(1)=PI/36.0
C     CALCULATE SURFACE DISPLACEMENT FROM 5 TO 85 DEGREES
      DO 100I=1,17
C     CALCULATE THE INCREMENT IN Y
      DELY=(2.0*PL)/(15.0*SIN(ALFA(I)))
C     ESTABLISH THE INITIAL ARRAY OF X AND Y FOR SURFACE
      X(16)=0.0
      Y(1)=0.0
      DO 200J=1,15
      X(J)=0.0
      Y(J+1)=Y(J)+DELY
  200 CONTINUE
      OLDX=0.0
      TIME(1)=-DELT
      WRITE(6,25)
   25 FORMAT(1H1,10X,'TIME',16X,'X',19X,'Y',18X,'FEE',//)
      DO 300K=1,60
C     CALCULATE THE ANGLE OF CORRECTION FEE
      DO 400L=1,16
      IF(L.EQ.1) GO TO 35
      IF(L.EQ.16) GO TO 45
      SLOPE=(4.0*X(L)-X(L+1)-3.0*OLDX)/(2.0*DELY)+(X(L+1)+OLDX-2.0*
     *X(L))/DELY
      GO TO 55
   35 SLOPE=(X(L+1)-X(L))/DELY
      GO TO 55
   45 SLOPE=(X(L)-OLDX)/DELY
      GO TO 55
   55 FEE=ATAN(SLOPE)
      IF(FEE.GT.0.0) FEE=0.0
C     CALCULATE THE CORRECTED ANGLE OF INCIDENCE GAMMA
      GAMMA=ALFA(I)+FEE
C     CALCULATE THE ANGLE OF REFLECTION BETA FOR REFLECTED TRANSVERSE
C     WAVE
      BETA=ARSIN(ST*SIN(GAMMA)/SL)
C     CALCULATE THE COEFFICIENT OF REFLECTION R
      R=(TAN(BETA)*(TAN(2.0*BETA))**2-TAN(GAMMA))/(TAN(BETA)*(TAN(2.0*
     *BETA))**2+TAN(GAMMA))
C     CALCULATE THE VELOCITY REFLECTION COEFFICIENT SR
      SR=((1.0+R)/(RO*ST))*((COTAN(2.0*BETA)*COS(BETA)+SIN(BETA))/
```

```
     *(SIN(2.0*BETA)))
C     CALCULATE THE SURFACE DISPLACEMENT DELX
      SIGMA=SIG*(1.0+(X(L)*COS(GAMMA)+Y(L)*SIN(GAMMA)-SL*(TIME(K)+
     *(DELT/2.0)))/PL)
      UP=SR*SIGMA
      DELX=UP*COS(2.0*BETA)*DELT
C     CALCULATE CHECK VALUES COM AND FIX
      COM=(X(L)*COS(GAMMA)+Y(L)*SIN(GAMMA))/SL
      FIX=(X(L)*COS(GAMMA)+Y(L)*SIN(GAMMA)+PL)/SL
C     CHECK TIME VARIABLE VS. ITS CHECK VALUES
      IF(TIME(K).LT.COM.OR.TIME(K).GT.FIX) DELX=0.0
      IF(DELX.LT.0.0) DELX=0.0
      OLDX=X(L)
      X(L)=X(L)+DELX
C     WRITE CALCULATED VALUES OF VARIABLES,TIME,X,Y,FEE
      WRITE(6,65) TIME(K),X(L),Y(L),FEE
   65 FORMAT(4E20.3)
  400 CONTINUE
      TIME(K+1)=TIME(K)+DELT
  300 CONTINUE
      ALFA(I+1)=ALFA(I)+DELA
  100 CONTINUE
      STOP
      END
```

```
C     SURFACE MOTION BY USING FIXED SURFACE ASSUMPTION
      DIMENSION TIME(2000),ALFA(20),X(18000),Y(18000)
C     READ FHYSICAL PARAMETERS OF MATERIAL
      READ(5,15) PI,PL,RO,SIG,SL,ST
   15 FORMAT(6E11.5)
C     INCREMENT IN TIME AND IN ANGLE OF INCIDENCE
      DELT=PL/(15.0*SL)
      DELA=PI/36.0
      ALFA(1)=PI/36.0
C     CALCULATE SURFACE DISPLACEMENT FROM 5 TO 85 DEGREES
      DO 100I=1,17
C     CALCULATE THE INCREMENT IN Y
      DELY=(2.0*PL)/(15.0*SIN(ALFA(I)))
C     ESTABLISH THE INITIAL ARRAY OF X AND Y FOR SURFACE
      X(16)=0.0
      Y(1)=0.0
      DO 200J=1,15
      X(J)=0.0
      Y(J+1)=Y(J)+DELY
  200 CONTINUE
      WRITE(6,25)
   25 FORMAT(1H1,10X,'TIME',16X,'X',19X,'Y',//)
C     CALCULATE THE ANGLE OF REFLECTION BETA FOR REFLECTED TRANSVERSE
C     WAVE
      BETA=ARSIN(ST*SIN(ALFA(I))/SL)
C     CALCULATE THE COEFFICIENT OF REFLECTION R
      R=(TAN(BETA)*(TAN(2.0*BETA))**2-TAN(ALFA(I)))/(TAN(BETA)*(TAN
     *(2.0*BETA))**2+TAN(ALFA(I)))
C     CALCULATE THE VELOCITY REFLECTION COEFFICIENT SR
      SR=((1.0+R)/(RO*ST))*((COTAN(2.0*BETA)*COS(BETA)+SIN(BETA))/
     *(SIN(2.0*BETA)))
C     CALCULATE CHECK VALUES COM AND FIX
      XA=SR*SIG*COS(2.0*BETA)
      TIME(1)=0.0
      DO 300L=1,60
      DO 400K=1,16
      YS=Y(K)*SIN(ALFA(I))
      COM=YS/SL
      FIX=COM+PL/SL
C     CHECK TIME VARIABLE VS. ITS CHECK VALUES
      IF(TIME(L).LT.COM.OR.TIME(L).GT.FIX) GO TO 45
C     CALCULATE THE SURFACE DISPLACEMENT DELX
      DELX=XA*(1.0+YS/PL-SL*0.5*(TIME(I)+COM)/PL)*(TIME(I)-COM)
      IF(DELX.LT.0.0) DELX=0.0
      X(K)=DELX
   45 CONTINUE
C     WRITE CALCULATED VALUES OF VARIABLES,TIME,X,Y
      WRITE(6,35) TIME(L),X(K),Y(K)
```

```
 35 FORMAT(3E20.3)
    Y(K+1)=Y(K)+DELY
400 CONTINUE
    TIME(L+1)=TIME(L)+DELT
300 CONTINUE
    ALFA(I+1)=ALFA(I)+DELA
100 CONTINUE
    STOP
```

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13. ABSTRACT

This work reports theoretical investigations of the surface motion in solids caused by reflection of an intense acoustic pulse at a pressure release surface done using the surface velocity reflection coefficient. A formula was derived for the surface velocity reflection coefficient which relates the surface particle velocity for an incident compression wave to the particle velocities of the incident and reflected waves.

To obtain the surface profiles, two different assumptions were used. The first assumption was the conventional one. In this assumption, surface motion was neglected during the period that the pulse strikes the surface and it is called the fixed surface approximation. The second assumption is to consider the motion and distortions of the surface and is called the moving surface approximation.

Four different materials were investigated and the results were compared for different materials and in the two different approximations.

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